Workshop in Convexity and High-Dimensional Probability
Short Talk Abstracts

Monday, May 23rd

Kendeda Buliding 152

An isoperimetric condition required for the Bezout inequality

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Maud Szusterman, Université Paris Diderot

In 2018, I. Soprunov and A. Zvavitch introduced a set of inequalities between mixed volumes, which they call Bezout inequalities, and which the simplex satisfies (ie with constant 1). All convex bodies satisfy a relaxed version of these inequalities, with constant at most n, as implied by Diskant inequality. It has been proven the simplex is the only minimizer among polytopes, and it was conjectured this characterization even holds among all convex bodies. Towards this conjecture, several necessary conditions for being a minimizer have been derived, excluding bodies with a strict point on their boundary, (weakly) decomposable bodies, or bodies with infinitely many facets. In this talk, I will present another excluding condition, of geometric flavour.

Isoperimetric Inequalities for Hessian Valuations

Jacopo Ulivelli, Sapienza University of Rome

We present a quick method to recover isoperimetric inequalities for integral functionals involving the gradient of convex functions and a boundary term, which under suitable conditions are valuations on convex functions with compact domain. This is achieved through Wulff's inequality and some correspondences between convex functions in $\mathbb{R}^n$ and convex bodies in $\mathbb{R}^{n+1}$.

Potential Theory with Multivariate Kernels

Damir Ferizović, KU Leuven

I will introduce the audience to the theory of minimization for energies with multivariate kernels, i.e. energies, in which pairwise interactions are replaced by interactions between triples or, more generally, n-tuples of particles. Appropriate analogues of conditionally positive definite kernels will be introduced, and I establish a series of relevant results in potential theory.

This is based on a paper with the same title, with D. Bilyk, A. Glazyrin, R. Matzke, J. Park and A. Vlasiuk.

Minimizing p-frame Energies and Mixed Volumes

Ryan W Matzke, Technische Universität Graz

There has been recent interest in the p-frame energies

$$I_p(\mu) = \int_{\mathbb{S}^d} \int_{\mathbb{S}^d} |\langle x, y \rangle|^p d\mu(x) d\mu(y), \qquad p \in (0, \infty),$$

for probability measures, μ, on the (real or complex) unit sphere $\mathbb{S}^d$. They have been shown to have important connections to quantum information theory and signal processing, as well as the mixed volumes of convex bodies and their projection bodies. In this talk, we will discuss how the minimizers of the p-frame energies produce minimizers of these mixed volumes.

The research in this presentation is joint work with Dmitriy Bilyk, Alexey Glazyrin, Josiah Park, and Oleksandr Vlasiuk.

Friday, May 27, 2022

Kendeda Buliding 230

On the framework of L_p summations for functions

Sudan Xing, University of Alberta

In the talk, the framework of L_p operations for functions including the $L_{p,s}$ convolution sum and the $L_{p,s}$ Asplund sum for functions when $p > 0$ will be presented. Particularly, the $L_{p,s}$ convolution summations contain the $L_{p,s}$ supremal-convolution when $p \geq 1$ and the $L_{p,s}$ inf-sup-convolution when $0 < p < 1$, respectively. Moreover, the $L_{p,s}$ Asplund summation is constructed by the L_p averages of bases for s-concave functions. Based on the properties of these summations for functions, we establish the L_p-Borell-Brascamp-Lieb inequalities for the $L_{p,s}$ supremal-convolution when $p \geq 1$. Furthermore, the integral formula for $L_{p,s}$ mixed quermassintegral in terms of the $L_{p,s}$ Asplund summation has also been discovered via tackling the variation formula of quermassintegral for functions when $p \geq 1$. This talk is based on the joint works with Dr. Michael Roysdon.

Problems in directional discrepancy

Michelle Mastrianni, University of Minnesota

The discrepancy of a point set in the unit cube provides a measure of how well-distributed the point set is. Precise behavior of the discrepancy strongly depends on the properties of the underlying collection of geometric test sets. In two dimensions, the discrepancy with respect to axis-parallel rectangles and rectangles rotated in arbitrary directions is well-understood. The increased complexity of the latter collection leads to discrepancy bounds of polynomial order, in contrast to logarithmic order in the axis-parallel case. In this talk we will examine the "in between" and discuss what happens when we restrict the allowed set of rotations to a smaller interval. In particular, we draw connections between this problem and recent work by Brandolini and Travaglini on the discrepancy with respect to particular classes of convex sets in the plane.

Moments of Gaussian quadratic forms with values in Banach space

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Rafał Meller, University of Warsaw

We will look for two-sided bounds (and upper bounds) for moments of Gaussian quadratic form given by $S = \sum a_{ij} g_i g_j$, where $(a_{ij})_{ij}$ is a Banach valued matrix. In particular, those bounds imply a version of Hanson-Wright inequality in Banach spaces. The talk will be based on joint work with Rafał Latała and Radosław Adamczak (Hanson-Wright inequality in Banach Spaces, Ann. Inst. Henri Poincaré Probab. Stat. 56 (2020), 2356-2376).

Extreme points of a subset of log-concave probability sequences

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Heshan Aravinda, University of Florida

Fradelizi & Guedon (2004) developed a localization result for s-concave measures in $\mathbb{R}^n$, and later presented a more general result for log-concave measures. Motivated by this work, Marsiglietti & Melbourne (2020) extended some of their results to one-dimensional discrete log-concave probabilities. More specifically, they proved that the extreme points of the set of log-concave probability sequences satisfying one linear constraint are log-affine.

One may also be interested in a generalized version of this result. More specifically, one would like to understand the shape of the extreme points of the set of discrete log-concave probabilities satisfying multiple constraints. It turns out that the extreme points are necessarily log-piecewise affine. In this talk, we will discuss this result.

Applications of sharp large deviation estimates to asymptotic convex geometry

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Yin-Ting Liao, Brown University

Random projections of high- dimensional probability measures have gained much attention recently in asymptotic convex geometry, high dimensional statistics and data science. Accurate estimation of tail probabilities is of importance in applications. Fluctuations of such objects are better understood, and a sufficient condition for approximately Gaussian fluctuations is the so-called thin-shell condition. Corresponding large deviation principles have been only more recently studied. More recently, techniques of sharp large deviation estimates have been introduced to study the refined large deviation estimates for random projections of high-dimensional measures. In this talk, I will describe an application of these refined estimates to studying volumetric problems in asymptotic convex geometry. This talk is based on several joint works with Kavita Ramanan.

Small ball probability of simple random tensor

Xuehan Hu, Texas A&M University

We study the small ball probability of the orthogonal projection of an order-d simple random tensor $X = X^{(1)} \otimes \cdots \otimes X^{(d)}$ onto an m-dimensional subspace F of $\mathbb{R}^{n^{\otimes d}}$, where $X^{(1)}, \cdots, X^{(d)}$ are independent random vectors in $\mathbb{R}^n$. If $X^{(1)}, \cdots, X^{(d)}$ are isotropic log-concave, or if $X^{(1)}, \cdots, X^{(d)}$ have independent coordinates with bounded densities, we give upper bounds of the small ball probability of the orthogonal projection of X onto F. This result is better than the known results when the dimension m of the subspace F is small or when the order d exceeds the dimension n of the vectors. This is based on work in progress jointly with G. Paouris.

On the L^p Aleksandrov problem for negative p

Stephanie Mui, New York University

Huang, Lutwak, Yang, and Zhang introduced the L^p integral curvature and posed the corresponding L^p Aleksandrov problem, the natural L^p extension of the classical integral curvature and Aleksandrov problem respectively. The problem asks about the existence of a convex body with prescribed L^p integral curvature measure. For the case of given even measures, the question will be solved for $p \in (-1, 0)$. Furthermore, a sufficient measure concentration condition will be provided for the case of $p \leq -1$, again provided that the given measure is even.