A NEW VERSION OF THE ISOMORPHIC BUSEMANN-PETTY PROBLEM FOR ARBITRARY FUNCTIONS

We consider the isomorphic Busemann-Petty problem for two different functions, as follows.

Theorem 1. *Let K, L be star bodies in $\mathbb{R}^n$, let $0 < k < n$, and let f, g be non-negative locally integrable functions on $\mathbb{R}^n$ so that $\|g\|_\infty = g(0) = 1$ and*

$$\int_{K\cap H} f \leq \int_{L\cap H} g \tag{1}$$

for all $(n-k)$-dimensional linear subspaces $H \subset \mathbb{R}^n$. Then

$$\int_K f \leq (d_{\mathrm{ovr}}(K, \mathcal{BP}_k^n))^k \frac{n}{n-k} |K|^{\frac{k}{n}} \left(\int_L g\right)^{\frac{n-k}{n}}.$$

Here $d_{\mathrm{ovr}}(K, \mathcal{BP}_k^n)$ is the outer volume ratio distance from the body K to the class of generalized k-intersection bodies in $\mathbb{R}^n$.

One advantage over previously known results is that the Banach-Mazur distance is replaced by smaller outer volume ratio distance. In particular, this allows to get an absolute constant in the case where K is an unconditional convex body. Also this version immediately implies the slicing inequality for arbitrary functions, similar to the case of volume.

This is joint work with Grigoris Paouris and Artem Zvavitch.