

A concise proof of the scalar B-conjecture

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Abstract

We give a direct scalar specialization of OpenAI’s moment-coordinate proof, including the tensor calculation, density argument, and limiting steps. The mechanism is a sharp variance bound for homogeneous convex energies: two complementary estimates contribute $1/q$ and $(q-1)/q$. We also prove its mixed-degree extension, an intersection inequality, and a gamma moment bound. Classical moment-map existence and regularity are the only substantive analytic inputs quoted without proof.

1 Introduction, statement, and history

Throughout, symmetry means symmetry about the origin. An even log-concave density is e^{-V} , where $V : \mathbb{R}^n \rightarrow (-\infty, +\infty]$ is proper, lower semicontinuous, even, and convex, with full-dimensional domain. A convex body is compact with nonempty interior.

Theorem 1 (Scalar B-inequality; OpenAI’s general case). *For every such V and every symmetric convex body K , the function*

$$t \longmapsto \mu(e^t K), \quad d\mu = e^{-V(x)} dx,$$

is log-concave. Equivalently, for $s, t \in \mathbb{R}$ and $0 \leq \lambda \leq 1$,

$$\mu(e^{(1-\lambda)s + \lambda t} K) \geq \mu(e^s K)^{1-\lambda} \mu(e^t K)^\lambda. \quad (1)$$

History. Banaszczyk proposed the Gaussian conjecture, popularized by Latała. Cordero-Erausquin, Fradelizi, and Maurey proved it in 2004, in the stronger coordinatewise Gaussian form; they also treated unconditional measures and sets [2]. Böröczky, Lutwak, Yang, and Zhang formulated log-Brunn–Minkowski and proved its planar symmetric case [3]. Saroglou’s transfer to even log-concave measures connected it to the B-conjecture [4]; Cordero-Erausquin and Rotem proved the latter for rotationally invariant log-concave measures [5]. OpenAI’s September 23, 2026 preprint proves the general logarithmic Brunn–Minkowski inequality by a moment-coordinate argument [1]. Here we extract the B-inequality directly, without using that geometric theorem as an intermediate step.

Ideas and attribution. The new analytic ingredient is OpenAI’s compensated tensor estimate (Lemma 2), coupled with its dense double-divergence test space. Its homogeneous variance mechanism and the resulting general B-inequality are credited to OpenAI, not claimed as independent discoveries here. The underlying coordinates come from Berman–Berndtsson and Cordero-Erausquin–Klartag [6, 7]; Klartag supplies their Hessian bound [8], and Fathi their Stein-kernel interpretation [9]. The transport-Hessian identity also appears in Kolesnikov–Livshyts–Rotem [10, Proposition 4.8]. The mixed-degree, intersection, and moment-generating formulations below are deductions developed in

our discussions, with full proofs and no claim of historical priority. Related human work on stronger variance inequalities includes [5, 10, 11].

The proof has three steps: construct the moment coordinates; obtain $\text{Var}(\psi) \leq \mathbb{E}\psi$; replace the indicator of K by a steep homogeneous barrier. Symmetry removes precisely the linear obstruction in the second step. Mere convexity in the original variables does not make the logarithmic dilation parameter jointly convex.

2 Preliminaries: moment coordinates

Initially let $Q \in C^\infty(\mathbb{R}^n)$ be even and $H = D^2Q \succeq \varepsilon I$, $\varepsilon > 0$, and put $d\nu = Z^{-1}e^{-Q}dx$. Expectations below refer to ν . The moment-map results [6, 7, 8] give a smooth even strictly convex ϕ such that

$$(\nabla\phi)_\#(e^{-\phi(z)}dz) = \nu, \quad \int e^{-\phi} = 1, \quad x = \nabla\phi(z), \quad \tau(x) = D_z^2\phi(z), \quad 0 \prec \tau \preceq \Lambda I, \quad (2)$$

where $\nabla\phi$ is a diffeomorphism and one may take $\Lambda = 4/\varepsilon$. The notation means that $X = \nabla\phi(Z_0)$ has law ν when Z_0 has density $e^{-\phi}$. Importantly, τ is a Hessian in the *source* variable, evaluated at $z = (\nabla\phi)^{-1}(x)$.

For precision, here is the passage from compact targets to (2). Apply compact moment-map existence and regularity [6, 7] to $\nu_R \propto e^{-Q}1_{B_R}dx$. Choose its moment potential ϕ_R even, with minimum at zero. Klartag's bound [8, Proposition 3.1 and Remark 3.5] is uniform in R . For $g_R = \phi_R - \phi_R(0)$, supporting planes at z and $-z$, followed by integration against $e^{-\phi_R}$, give

$$c|y| - n \leq \mathbb{E}_{\nu_R}|X \cdot y| - n \leq g_R(y) \leq \frac{\Lambda}{2}|y|^2.$$

Here $\int z \cdot \nabla\phi_R e^{-\phi_R}dz = n$ by a cutoff integration; compact target support gives the source exponential tails needed for it. The constant $c > 0$ is uniform for large R , since the target densities have a common positive lower bound on B_1 . Normalization, $e^{\phi_R(0)} = \int e^{-g_R}$, bounds $\phi_R(0)$. The moment equation

$$Q(\nabla\phi_R) = \phi_R + \log \det D^2\phi_R - \log Z_R \quad (3)$$

then bounds the determinant below on each fixed source ball. The upper Hessian bound gives local uniform ellipticity; interior Monge–Ampère regularity and Schauder bootstrapping give locally smooth convergence of a subsequence. Exponential domination passes normalization and the pushforward identity to its limit. Its gradient image is dense since ν has positive density everywhere. Given x_0 , finitely many image points surrounding x_0 have supporting planes making $\phi(z) - x_0 \cdot z$ coercive. Its minimum realizes $\nabla\phi = x_0$. This proves surjectivity and all the stated whole-space properties. This exhaustion follows the analytic setup of [1]; see also [12, Lemma 3.1].

Write $M = \tau^{-1}$ and let $\delta v = \nabla Q \cdot v - \text{div } v$; on matrices δ acts row by row. Source integration by parts gives the Stein identity and its operator form:

$$\mathbb{E}[Xa(X)] = \mathbb{E}[\tau \nabla a(X)], \quad Af := \delta(\tau \nabla f) = x \cdot \nabla f - \tau : D^2f. \quad (4)$$

Indeed, integrate $\phi_{z_i} a(\nabla\phi) e^{-\phi}$ in z_i . Thus A is symmetric on compact tests, $\mathbb{E}gAf = \mathbb{E}\nabla g \cdot \tau \nabla f$, and $A1 = 0$, $Ax_i = x_i$. Here $B : C = \text{tr}(B^T C)$; all unmarked derivatives are in x .

3 The analytic core

For $f \in C_c^\infty(\mathbb{R}^n)$ set

$$h = D_x^2 f, \quad B = \tau h \tau, \quad w = \delta B, \quad u = \delta w. \quad (5)$$

Direct differentiation and two adjunctions yield

$$w = \nabla_z(A - 1)f = \tau \nabla_x(A - 1)f, \quad u = A(A - 1)f, \quad (6)$$

$$\mathbb{E}ug = \mathbb{E}w \cdot \nabla g = \mathbb{E}B : D^2g, \quad (7)$$

$$\mathbb{E}(Af)^2 - \mathbb{E}fAf = \mathbb{E}\|\tau^{1/2}D^2f\tau^{1/2}\|_{\text{HS}}^2. \quad (8)$$

For example, with $C_{ijk} = \phi_{z_i z_j z_k}$, $w_a = x_k \tau_{ai} f_{x_i x_k} - C_{aik} f_{x_i x_k} - \tau_{ai} \tau_{jk} f_{x_i x_k x_j}$, proving (6). Equation (8) follows by pairing ∇f with w . These are OpenAI's adjoint identities [1, Section 4]; (8) is also a specialization of [10, Proposition 4.8].

Lemma 2 (OpenAI's compensated tensor estimate). *For the fields (5),*

$$\mathbb{E}u^2 \geq \mathbb{E}w^T H w + \mathbb{E} \text{tr}(MBHB). \quad (9)$$

Proof. We give the square completion of [1, Lemma 6.1]; the same calculation is reproduced in [12, Section 3.1]. Weighted integration by parts first gives

$$\mathbb{E}u^2 = \mathbb{E}w^T H w + \mathbb{E} \text{tr}((D_x w)^2). \quad (10)$$

The last trace is a matrix-square trace, not a squared entrywise norm. Use $\partial_i = \partial_{z_i}$, $\partial_i^* = x_i - \partial_i$, $(C_i)_{jk} = C_{ijk}$, $P = \tau h$, and $T = h\tau h$. Differentiating (3) twice gives

$$(\tau H \tau - \tau)_{ij} = (\partial_d - x_d)(M_{dk} C_{ijk}) - \text{tr}(M C_i M C_j). \quad (11)$$

To check it, the first derivative is $\tau_{ik} Q_{x_k} = x_i + \text{tr}(M C_i)$, and $\partial_d M_{dk} - x_d M_{dk} = -Q_{x_k}$. Repeated indices are summed.

Since $w_a = \partial_k^* P_{ak}$ and $[\partial_l, \partial_k^*] = \tau_{kl}$, the symmetric matrix $R = D_z w - B$ satisfies $R_{al} = \partial_k^* \partial_l P_{ak}$. Expand $D_x w = (B + R)M$ and integrate the cross term. Separately contract (11) with T . The two results are

$$\begin{aligned} \mathbb{E} \text{tr}((D_x w)^2) &= \mathbb{E} \text{tr}(MBMB) + 2\mathbb{E}(\partial_k h_{al})(\partial_l P_{ak}) + \mathbb{E} \text{tr}(MRMR), \\ \mathbb{E} \text{tr}(MBHB) &= \mathbb{E} \text{tr}(MBMB) - \mathbb{E}(\partial_d T_{ij}) M_{dk} C_{ijk} - \mathbb{E} T_{ij} \text{tr}(M C_i M C_j). \end{aligned}$$

All integrations involve compact fields. Their contractions are invariant under constant dual changes $z = Lz'$, $x' = L^T x$. At a fixed point take $L = \tau^{-1/2}$; hence $\tau = I$ there, *after* differentiating. Put $S_{ijk} = f_{x_i x_j x_k}$ and $J_{ijk} = h_{ia} C_{ajk}$ in these normalized coordinates. The three derivative contractions in the difference above are respectively

$$2|S|^2 + 2\langle S, J \rangle, \quad 2\langle S, J \rangle + \langle J, J^{(12)} \rangle, \quad |J|^2, \quad J_{ijk}^{(12)} = J_{jik}.$$

As S is fully symmetric and J is symmetric in its last two indices, full symmetrization is $(\text{Sym } J)_{ijk} = (J_{ijk} + J_{jik} + J_{kij})/3$; thus $3|\text{Sym } J|^2 = |J|^2 + 2\langle J, J^{(12)} \rangle$ and $\langle S, \text{Sym } J \rangle = \langle S, J \rangle$. Therefore

$$\mathbb{E}u^2 - \mathbb{E}w^T H w - \mathbb{E} \text{tr}(MBHB) = \mathbb{E}\left[\|M^{1/2} R M^{1/2}\|_{\text{HS}}^2 + 2|S + \text{Sym } J|^2 + \frac{1}{6}|J - J^{(12)}|^2\right] \geq 0. \quad (12)$$

The normalized tensor expressions define invariant scalars; no varying frame is differentiated. This proves (9). $\square$

Lemma 3 (Density of even tests; OpenAI). *The functions $A(A - 1)f$, for compact smooth even f , are dense in $L_{\text{even},0}^2(\nu)$, the even functions of mean zero.*

Proof. We include the cutoff argument of [1, Lemma 5.1]. Let $g \in L^2_{\text{even},0}$ annihilate this range. Parity and symmetry give $A(A-1)g = 0$ distributionally. Its principal symbol $(\xi^T \tau \xi)^2$ is locally elliptic, so interior elliptic regularity makes g smooth. Choose smooth radial $0 \leq \eta_r \leq 1$, one on B_r and zero outside B_{2r} , with first and second derivatives $O(r^{-1})$, $O(r^{-2})$. Writing $\Gamma(a, b) = \nabla a \cdot \tau \nabla b$, boundedness of τ gives $\Gamma(\eta_r, \eta_r) \leq C/r^2$ and $|A\eta_r| \leq C$. Set

$$X_r = \|\eta_r^2 Ag\|_2, \quad Y_r^2 = \mathbb{E} \eta_r^2 \Gamma(g, g), \quad G_0 = \|g\|_2.$$

The product rule $A(ab) = aAb + bAa - 2\Gamma(a, b)$ gives

$$(A-1)(\eta^4 g) = \eta^4 Ag + g\{4\eta^3 A\eta - 12\eta^2 \Gamma(\eta, \eta) - \eta^4\} - 8\eta^3 \Gamma(\eta, g).$$

Testing the equation against $\eta_r^4 g$, and separately integrating $\mathbb{E} \eta_r^2 g Ag$, yields by Cauchy–Schwarz

$$X_r \leq CG_0 + (C/r)Y_r, \quad Y_r^2 \leq G_0 X_r + (C/r)G_0 Y_r.$$

These bounds are uniform, so $v = Ag \in L^2$. Now $Av = v$ and $A(g-v) = 0$. Testing the latter with $\eta_r^2(g-v)$ shows that $g-v$ is constant. Testing the former with $\eta_r^2 v$ gives $\mathbb{E} \Gamma(v, v) < \infty$. For $f_r = \eta_r v$, the product rule shows $f_r \rightarrow v$, $Af_r \rightarrow v$ in L^2 : $vA\eta_r$ tends to zero by its annular L^2 tail and $\|\Gamma(\eta_r, v)\|_2 \leq Cr^{-1}(\mathbb{E} \Gamma(v, v))^{1/2}$. In (8) the left side therefore tends to zero. On each fixed ball $f_r = v$ eventually, hence $D^2 v = 0$. Thus g is affine, and its evenness and zero mean force $g = 0$. $\square$

4 The sharp variance estimate

Proposition 4 (Mixed homogeneous energies). *Let U be smooth and even, with $D^2 U \succeq \varepsilon I$, and let $\psi_i \not\equiv 0$, $1 \leq i \leq m$, be even convex homogeneous polynomials of degrees $q_i \geq 2$. For $d\nu \propto e^{-U - \sum_i \psi_i} dx$ and $a_i = \mathbb{E} \psi_i$,*

$$\text{Var}_\nu \left(\sum_i b_i \psi_i \right) \leq \sum_i b_i^2 a_i \quad (b \in \mathbb{R}^m), \quad \text{Cov}_\nu(\psi_1, \dots, \psi_m) \preceq \text{diag}(a_1, \dots, a_m). \quad (13)$$

In particular, $\text{Var}_\nu \psi \leq \mathbb{E} \psi$ for one energy.

Proof. Apply the preceding lemmas to $Q = U + \sum_i \psi_i$. For a compact even test put $d_i = \mathbb{E} u \psi_i$ and $G_i = D^2 \psi_i \succeq 0$. Euler’s identity, (4), and (7) give

$$G_i x = (q_i - 1) \nabla \psi_i, \quad \mathbb{E} \text{tr}(\tau G_i) = q_i a_i, \quad \mathbb{E} x^T G_i x = q_i(q_i - 1) a_i, \quad d_i = \mathbb{E} \text{tr}(B G_i) = \mathbb{E} w \cdot \nabla \psi_i.$$

The noncompact Stein test is justified by cutoffs: τ is bounded, the tests have polynomial growth, and ν has Gaussian tails. Matrix and vector Cauchy–Schwarz give, respectively,

$$\mathbb{E} \text{tr}(M B G_i B) \geq \frac{d_i^2}{q_i a_i}, \quad \mathbb{E} w^T G_i w \geq \frac{(q_i - 1) d_i^2}{q_i a_i}. \quad (14)$$

For the first pair $G_i^{1/2} B M^{1/2}$ with $G_i^{1/2} \tau^{1/2}$; for the second pair $G_i^{1/2} w$ with $G_i^{1/2} x$. This works even if G_i is singular. Since $H \succeq \sum_i G_i$, (9) and (14) imply

$$\mathbb{E} u^2 \geq \sum_i \frac{d_i^2}{a_i}, \quad \left(\mathbb{E} u \sum_i b_i \psi_i \right)^2 \leq \left(\sum_i b_i^2 a_i \right) \mathbb{E} u^2.$$

Use Lemma 3 to let $u \rightarrow \sum_i b_i(\psi_i - a_i)$ in L^2 . The result follows. The decisive cancellation is $1/q_i + (q_i - 1)/q_i = 1$, separately for every degree. $\square$

The common-degree powers-of-linear-forms case is OpenAI’s variance mechanism [1, Proposition 2.1 and Section 7]. The proof above allows the degrees to differ and the homogeneous polynomials to be arbitrary convex ones. It uses the tensor estimate, not an assumed universal strengthening of the Brascamp–Lieb inequality.

5 Proof of the B-inequality and the intersection extension

Proof of Theorem 1. First suppose V is smooth with $D^2V \succeq \varepsilon I$, and $K = \{x : |\ell_j(x)| \leq 1, 1 \leq j \leq N\}$ is a bounded symmetric polytope. For even integers q define

$$\psi_q(x) = \sum_{j=1}^N \ell_j(x)^q, \quad Z_q(t) = \int_{\mathbb{R}^n} e^{-V(x) - e^{-qt}\psi_q(x)} dx.$$

Apply Proposition 4 to $\psi = e^{-qt}\psi_q$. Differentiating under the integral is valid locally uniformly in t , by Gaussian domination with polynomial factors, and yields

$$(\log Z_q)''(t) = q^2 (\text{Var}_{\nu_{q,t}} \psi - \mathbb{E}_{\nu_{q,t}} \psi) \leq 0. \quad (15)$$

Off $\partial(e^t K)$, $e^{-e^{-qt}\psi_q} \rightarrow 1_{e^t K}$ as $q \rightarrow \infty$ through even integers. Dominated convergence and the three-point log-concavity inequality imply (1) for K . Decreasing outer symmetric polytopes then give every convex body.

For general V , subtract $V(0)$ so $V \geq 0$. Its Moreau envelopes $V^{[j]}(x) = \inf_y \{V(y) + j|x - y|^2/2\}$ increase to V . Convolve them with smooth even nonnegative mollifiers of radius tending to zero, chosen so that the error on B_j is at most $1/j$, and add $|x|^2/(2j)$. The resulting V_j are smooth, even, uniformly convex, nonnegative, and converge pointwise to V . On every bounded dilate of K , $e^{-V_j} \leq 1$. Dominated convergence passes (1) to V . $\square$

The same mechanism retains independent scale parameters.

Corollary 5 (Intersection inequality). *For symmetric convex bodies $K_1, \dots, K_m$ and any μ as in Theorem 1,*

$$(t_1, \dots, t_m) \mapsto \log \mu \left(\bigcap_{i=1}^m e^{t_i} K_i \right) \quad \text{is concave on } \mathbb{R}^m. \quad (16)$$

Proof. For polytopes use one barrier $\psi_{i,q}$ for each body and set $Y_i = e^{-qt_i}\psi_{i,q}$. The Hessian of $\log \int e^{-V - \sum_i Y_i}$ is $q^2 [\text{Cov}(Y) - \text{diag}(\mathbb{E} Y_i)] \preceq 0$ by Proposition 4. The same barrier limit, outer approximations, and regularization of V prove (16). $\square$

This is a geometric reformulation of the logarithmic inequality in [1], rather than a priority claim for a stronger theorem. The parameters scale entire constraints. They do not assert the coordinatewise strong B-property for an arbitrary even log-concave measure.

6 Two further consequences

Corollary 6 (Infinitesimal B-inequality). *Let V be smooth, even and convex, and let ν_K be e^{-V} normalized on a symmetric convex body K . With $\mathcal{D} = x \cdot \nabla$,*

$$\text{Var}_{\nu_K}(\mathcal{D}V) \leq \mathbb{E}_{\nu_K} \mathcal{D}^2 V = \mathbb{E}_{\nu_K} [x \cdot \nabla V + x^T \mathcal{D}^2 V x]. \quad (17)$$

If V is homogeneous of degree $q > 0$, this reads $\text{Var}_{\nu_K}(V) \leq \mathbb{E}_{\nu_K} V$.

Proof. Write $\mu(e^t K) = e^{nt} \int_K e^{-V(e^t x)} dx$. Twice differentiating its logarithm at zero gives exactly $\text{Var}_{\nu_K}(\mathcal{D}V) - \mathbb{E}_{\nu_K} \mathcal{D}^2 V \leq 0$. For homogeneous V , $\mathcal{D}V = qV$ and $\mathcal{D}^2 V = q^2 V$. $\square$

Conversely, (17) for every dilate of K gives Theorem 1 for smooth V . This connects the proof to the restricted variance questions in our discussions; it does not prove a variance inequality for every even test function of the type considered in [11].

Corollary 7 (Gamma bound). *Under the assumptions of Proposition 4, for all real $\theta_i < 1$,*

$$\mathbb{E} \exp \left(\sum_i \theta_i \psi_i \right) \leq \prod_i (1 - \theta_i)^{-a_i}. \quad (18)$$

Proof. Set $\Phi(s) = \log \int e^{-U - \sum_i e^{s_i} \psi_i} dx$. Differentiation and (13) give $D^2 \Phi \preceq 0$ and $\nabla \Phi(0) = -(a_i)_i$. Its tangent-plane bound $\Phi(s) - \Phi(0) \leq -\sum_i a_i s_i$, applied at $s_i = \log(1 - \theta_i)$, is (18). $\square$

The bound is the moment-generating function of independent rate-one gamma variables with shapes a_i ; independence of the actual energies is not assumed. Its constant is sharp in the limit $U \downarrow 0$: for $\psi_i(x) = |x_i|^{q_i}$ the energies are independent $\text{Gamma}(1/q_i, 1)$ variables by the change of variables $y_i = |x_i|^{q_i}$.

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