

Schedule for the Topics Course “Concentration of Measure Phenomenon and Convexity” Fall 2023

Below please find the outline of the course. Topics marked with a star are likely to pop up as optional home works or be postponed till the Spring 2024 course in High-Dimensional Probability. Students are invited to give input and make requests of topics. This schedule is subject to changes throughout the semester. The papers and surveys linked in the third column are clickable; you may also find a long list of recommended books on the course webpage.

Also see [here](#) the schedule of the “second part” (High-dimensional Probability taught in Spring 2024)

Topic	Sub-topics	Resources	# of lectures
Laplace method	<ol style="list-style-type: none"> 1. Gaussian integral 2. Area of the sphere 3. Volume of the Ball 4. Concentration on the ball via Laplace method; concentration the sphere (statement) 5. Also the thin shell concentration for the ball; also for any convex body via the general polar system 	<p>Book: DeBrujn, Asymptotic methods in Analysis</p> <p>Book: Alex Iosevich, View from the Top</p>	1
Basic Convexity, Brunn-Minkowski theory, Prekopa-Leindler inequality	<ol style="list-style-type: none"> 1. Basic concepts from convexity 2. Brunn-Minkowski inequality (Lyusternik’s proof via induction; symmetrization proof) 3. The isoperimetric inequality 4. Steiner symmetrization, proof via Steiner 5. Mixed volumes and Minkowski’s first inequality 6. Brunn’s concavity principle 7. Log-concave measures 8. Prekopa-Leindler inequality via tensorisation 9. Log-concavity of marginals and convolution 10. Borell-Brascamp-Lieb inequality 11. Steiner formula* 12. Rogers-Shepherd inequality* 13. Uniqueness in the Minkowski problem* 14. Minkowski second inequality* 15. Grunbaum’s inequality* 16. Urysonn’s inequality* 17. Busemann’s theorem* 	<p>Book: Artstein-Avidan, Giannopolous, Milman, Asymptotic Geometric Analysis part 1</p> <p>Tao</p> <p>Klartag</p>	6

Topic	Sub-topics	Resources	# of lectures
Duality and the linearization method, Isoperimetric and Functional Inequalities	<ol style="list-style-type: none"> 1. Legendre transform and its properties, relation to Prekopa (motivated) 2. Generalized Log-Sobolev inequality via Prekopa-Leindler; functional Minkowski first; corollaries/ reformulations; Lebesgue vs Gaussian; Sobolev; Beckner 3. Brascamp-Lieb inequality from Generalized Log-Sobolev 4. Gaussian Poincare inequality; related discussion about Poincare inequalities, Poincare on the circle; Poincare inequality – the basics; statement of the Payne-Weinberger inequality 5. A dimensional version of Log-Sobolev and Sobolev inequalities via an extended BBL 6. A dimensional extension of Brascamp-Lieb inequality 	<p>Bobkov, Ledoux</p> <p>Cordero-Erasquin</p> <p>Cordero-Erasquin</p> <p>Klartag</p>	<p>3</p>

Topic	Sub-topics	Resources	# of lectures
Blaschke-Santalo inequality and its linearization	<ol style="list-style-type: none"> 1. Blaschke-Santalo inequality; proof via Steiner symmetrizations 2. Relation to -2-Beckner on the circle; phi-entropy 3. Functional Blaschke-Santalo inequality 4. Symmetric Gaussian Poincare inequality on the space via Blaschke-Santalo 5. An excursion into mass transport 6. The B-theorem, the B-conjecture, the B-conjecture for the unconditional case 7. Reverse Log-Sobolev inequality 8. Transport-entropy inequalities of Talagrand and Fathi 	<p>Book: Artstein-Avidan, Giannopolous, Milman, Asymptotic Geometric Analysis part 1</p> <p>Klartag</p> <p>Klartag</p> <p>Book: Villani, Topics in Optimal Transportation</p> <p>Cordero-Erasquin, Fradelizi, Maurey</p> <p>Kolesnikov</p> <p>McCann</p> <p>Ball</p> <p>Artstein-Avidan, Klartag, Milman</p> <p>Cordero-Erasquin</p> <p>Cordero-Erasquin</p> <p>Gozlan, Leonard</p> <p>Gozlan, Fradelizi, Sadovsky, Zugmeyer</p> <p>Klartag</p> <p>Fathi</p> <p>Artstein-Avidan, Klartag, Schutt, Werner</p> <p>Calgar, Fradelizi, Guedon, Lehec, Schutt, Werner</p>	<p>5</p>

Topic	Sub-topics	Resources	# of lectures
Concentration of measure phenomenon via Brunn-Minkowski and isoperimetry; soft approach	<ol style="list-style-type: none"> 1. Isomorphic concentration on the sphere 2. Isomorphic Gaussian Concentration 3. Concentration function 4. Concentration for Lipschitz functions, with examples on the sphere 5. Borell's Inequality via Prekopa-Leindler, moment comparisons 6. Concentration via Laplace functional 7. Thin shell etc 8. Thin shell unconditional 9. Paouris inequality 10. Herbst argument via Log-Sobolev 11. Exponential concentration via Poincare inequality 12. Klartag-Vershynin's small ball estimate via the B-theorem 13. Concentration via Talagrand 14. Concentration on the Hamming cube* 	<p>Book: Artstein-Avidan, Giannopolous, Milman, Asymptotic Geometric Analysis part 1</p> <p>Book: Artstein-Avidan, Giannopolous, Milman, Asymptotic Geometric Analysis part 2</p> <p><u>Gozlan-Leonard</u></p> <p><u>Klartag-Vershynin</u></p> <p><u>Naor</u></p>	4

Topic	Sub-topics	Resources	# of lectures
Gaussian Measure, related isoperimetric and functional inequalities, and the Gaussian concentration	<ol style="list-style-type: none"> 1. Ehrhard symmetrization 2. Ehrhard inequality via symmetrization 3. Gaussian isoperimetric inequality via Ehrhard 4. Estimate for the isoperimetry on the hypercube 5. Functional Ehrhard inequality 6. Gaussian concentration via isoperimetry 7. Bobkov's inequality via Ehrhard 8. Ehrhard-Brascamp-Lieb 9. The Gaussian barycenter inequality 10. Various additional Gaussian inequalities 11. The (non-symmetric) Gaussian dilation inequality plus upper bound 12. The S-inequality 13. An improved Gaussian Poincare inequality for linear functions 14. Gaussian Isoperimetric inequality via Bobkov's inequality* 15. Gaussian correlation inequality* 16. Khatri-sidak* 17. Gaussian measure is the "best" among isotropic log-concave 	<p>Book: Bogachev, Gaussian measures</p> <p>Ledoux</p> <p>Neeman</p> <p>Bobkov</p> <p>Latala</p> <p>Latala, Oleszkiewicz</p> <p>Book: Artstein-Avidan, Giannopolous, Milman, Asymptotic Geometric Analysis part 1</p> <p>Book: Artstein-Avidan, Giannopolous, Milman, Asymptotic Geometric Analysis part 2</p>	5

Topic	Sub-topics	Resources	# of lectures
The L2 method	<ol style="list-style-type: none"> 1. Review of existence results from PDE and Functional Analysis, integration by parts 2. Bochner's identity (2 proofs) 3. The L2 proof of the Brascamp-Lieb inequality 4. Brascamp-Lieb with a concave function 5. The L2 proof of the B-theorem; further connections and conjectures 6. Klartag's H-1 inequality 7. Klartag's strengthening of the Lichnerowich's inequality 8. The B-theorem for rotation-invariant measures 9. Kolesnikov-Livshyts theorem* 10. Cordero-Erasquin-Rotem theorem and their extension of the Eskenadis-Moschidis result* 11. Livshyts lower bound on the dimensional Brunn-Minkowski* 	<p>Klartag</p> <p>Klartag</p> <p>Barthe, Klartag</p> <p>Klartag</p> <p>Cordero-Erasquin, Klartag</p> <p>Cordero-Erasquin, Fradelizi, Maurey</p> <p>Kolesnikov, Milman</p> <p>Kolesnikov, Livshyts</p> <p>Livshyts</p> <p>Cordero-Erasquin, Rotem</p>	4