Schedule for the Topics Course ``High-dimensional Probability” Spring 2024

Below please find the outline of the course. Topics marked with a star are likely to pop up as optional home works. Students are invited to give input and make requests of topics. This schedule is subject to changes throughout the semester. The papers and surveys linked in the third column are clickable; you may also find a long list of recommended books on the course webpage.

Here find also the schedule for the “preceding” semester (Concentration of Measure Phenomena Topics Course that was taught Fall 2023).

Topic	Sub-topics	Literature	Number of lectures
Probabilistic method and some fun facts	1. Introduction and preliminaries 2. A strip centered at a point in the cube via random rounding 3. Randomized Caratheodory teorem 4. Application to covering of polytopes 5. Standard epsilon-net argument 6. Efficient net-argument via random rounding 7. Geometric meaning of CLT, some pretty pictures, High-dimensional phenomenon	Vershynin, High-Dimensional Probability Klartag, Livshyts Livshyts	3

Topic	Sub-topics	Literature	Number of lectures
Concentration inequalities for sums of independent random variables, high-dimensional phenomena stemming from independence	1. Hoeffding's inequality 2. Application to random graphs 3. Chernoff's inequality 4. Sub-Gaussian random variables 5. Khinchine inequality 6. Sub-exponential random variables 7. Bernstein inequality 8. Norm of a sub-Gaussian random vector 9. Grothendick's inequality 10. Azuma*	Vershynin, High-Dimensional Probability Book: Artstein-Avidan, Giannoplolous, Milman, Asymptotic Geometric Analysis part 1	5
Random matrices	1. Random matrices 2. Norm of a sub-Gaussian random matrix 3. Matrix Bernstein Inequality 4. Tensorization and the Levy concentration function, small ball inequalities 5. Smallest singular value of tall matrices (general, in expectation) 6. Smallest singular value of tall Wigner matrices whp 7. Smallest singular value of sub-Gaussian iid random matrices (with polynomial bound on the probability) 8. More general case	Vershynin, High-Dimensional Probability Rudelson, Vershynin Rudelson, Vershynin Rudelson, Vershynin Rudelson Rudelson Livshyts Livshyts, Tikhomirov, Vershynin Tikhomirov	5

Topic	Sub-topics	Literature	Number of lectures
Gaussian Random Processes and their suprema	1. Basic concepts, definitions 2. Slepian's inequality 3. Sudakov-Fernique inequality 4. Gordon inequality* 5. Gaussian random matrices 6. Sudakov minoration inequality* 7. Gaussian width, random projections*	Vershynin, High-Dimensional Probability	2
The semigroup method	1. Background on the Markov semi-groups 2. The Ornstein-Uhlenbeck semigroup, hypercontractivity, semi-group proof of the Gaussian Poincare inequality 3. Discussion on Poincare inequalities 4. Semi-group proof of Log-Sobolev inequality 5. P-Beckner inequalities* 6. Phi-entropy inequalities* 7. Semi-group proof of the Ehrhard inequality* 8. Semi-group proof of the Bobkov inequality 9. Bobkov's inequality via 2-point symmetrizations*	Book: Bakry, Ledoux, Gentil, Analysis and Geometry of Markov diffusion operators Book: Artstein-Avidan, Giannoplolous, Milman, Asymptotic Geometric Analysis part 2 Borell van Handel's lecture notes Carlen, Kerce Barthe, Huet	5

Topic	Sub-topics	Literature	Number of lectures
Applications to concentration of measure	1. Gaussian isoperimetry and Gaussian concentration 2. Gaussian and spherical concentration for Lipschitz functions 3. Johnson-Lindenstraus Lemma 4. Tails of norms of Gaussian random matrices* 5. Dvoretzky theorem* 6. EPI* 7. More on information-theoretic inequalities*	Book: Artstein-Avidan, Giannoplolous, Milman, Asymptotic Geometric Analysis part 1 Book: Artstein-Avidan, Giannoplolous, Milman, Asymptotic Geometric Analysis part 2 Vershynin’s book	2

Topic	Sub-topics	Literature	Number of lectures
Methods of Mass transport	1. Monge problem, dual problem, disintegration of measure 2. Rockafellar theorem 3. Brenier theorem 4. Ball's proof of the Brunn-Minkowski inequality via mass transport 5. Isoperimetric inequality 6. Mass transport proof of Prekopa-Leindler inequality 7. Log-concave random vectors are sub-exponential* 8. Gromov's proof of the isoperimetric inequality* 9. Cafarelli's contraction theorem 10. Application: extending Poincare and Log-Sobolev to strongly log-concave measures 11. Talagrand's transport-entropy inequality*	Klartag Klartag Book: Artstein-Avidan, Giannoplolous, Milman, Asymptotic Geometric Analysis part 1 Book: Artstein-Avidan, Giannoplolous, Milman, Asymptotic Geometric Analysis part 2 Book: Villani, Topics in Optimal Transportation Book: Figalli, Glaudo, An Invitation to Optimal Transport, Wasserstein Distances, and Gradient Flows. Cordero-Erasquin, Fradelizi, Maurey Kolesnikov McCann Ball	4

Topic	Sub-topics	Literature	Number of lectures
Convex Localization and application to concentration	1. Basics of convex localization, discussion of eigenfunctions and energy minimization 2. Proof of the Payne-Weinberger inequality 3. The four function theorem 4. Diameter of isotropic convex body 5. Gromov's waist inequality* 6. Nazarov-Sodin* 7. Brunn-Minkowski inequality via the ``mass transport" localization*	Klartag Klartag KLS Book: Artstein-Avidan, Giannoplolous, Milman, Asymptotic Geometric Analysis part 1 Vershynin, High-Dimensional Probability	4
Concentration of measure via Stochastic Calculus*	1. Basics of the Stochastic Calculus* 2. Gaussian Poincare via Stochastic Calculus* 3. Gaussian concentration via Stochastic Calculus* 4. Borell's noise stability 5. Background on Stochastic Localization* 6. Klartag's bound on the KLS*	Eldan ICM notes Klartag Lee, Vempala Book: Oksendal, Stochastic Differential Equations	4

Some additional/alternative topics:

Topic	Details
Asymptotic Analysis, log-concave measures, Paouris's inequality, Klartag's CLT for convex sets	
Non-asymptotic theory of random matrices*	
Slicing problem and related questions, Klartag's proof of the best KLS bound*	
Concentration on the Hamming cube*	
Reverse isoperimetry and Multi-dimensional CLTs*	
Isomorphic theory	1. John's position, Banach-Mazur distance, positions of convex sets and isotropicity* 2. Reverse isoperimetric inequalities 3. Maximal surface area position
Ball's form of Borell-Brascamp-Lieb inequality, cube slicing and the volume ratio bound*	
Dvoretzky's theorem, Milman's proof	
MM* estimates	
Bounds for the illumination conjecture	
Information theory, EPI	
Radon transform, Busemann-Petty problem, Shephard problem	
Functional inequalities for vector-valued functions	
Concentration for convex functions, super-concentration, Talagrand/Cordero-Erasquin's inequality	
Gaussian correlation inequality	
Rearrangement inequalities and methods	