HOME WORK I, ANALYSIS II

Due January 23. Problems marked with asterisk are optional, but highly recommended. Please contact me if you have any questions!

1. Suppose f and g are Riemann-integrable on an interval I. Show that $f + g$ is also Riemann-integrable on the interval I.

2. Write the formal proof for the First Fundamental Theorem of Calculus which we sketched in class.

3. Write out the formal proof for the integration-by-parts identity, stated in class.

4. Suppose that $f : [a, b] \to \mathbb{R}$ is monotone increasing. Show that for any $x \in [a, b]$, the function $F(x) = \int_a^x f(y)dy$ is differentiable at x if and only if it is continuous at x.

5. Let $\alpha(x) = sgn(x)$. Show that any continuous function $f : [-1, 1] \to \mathbb{R}$ is Stieltjes-Riemann integrable with respect to this α. Find $\int_{-1}^{1} f(x)d\alpha(x)$.

6^*. Is a composition of Riemann-integrable functions necessarily Riemann-integrable?

7^*. a) Can a function be Riemann-integrable on an interval, but not have a primitive? b) Can a function have a primitive on a closed interval, but not be Riemann-integrable?

(recall that a function has a primitive if its antiderivative is differentiable and its derivative equals to f).