HOME WORK II, ANALYSIS I

Due September 19.

1. Construct an explicit bijection between sets $A \times B$ and $B \times A$.

2. Let A, B, C be sets (not necessarily finite!). Show that the sets $(A^B)^C$ and $A^{B \times C}$ have the same cardinality.

3. Let A, B, C be finite sets. Show that
a) For any $a \notin A$ we have $card(A \cup \{a\}) = card(A) + 1$;
b) If $A \subset B$ then $card(A) \leq card(B)$;
c) $card(A \cup B) \leq card(A) + card(B)$;
c') $card(A \cup B) = card(A) + card(B)$ if and only if $A \cap B = \emptyset$;
d) $card(A \times B) = card(A)card(B)$.

4. Let $A_1, ..., A_n$ be finite sets such that $card\left(\cup_{i=1}^{n} A_i\right) \geq n + 1$. Show that for at least one $i \in \{1, ..., n\}$ we have $card(A_i) \geq 2$.

5. Recall that we say that $card(A) \leq card(B)$ if there exists an injection from A to B. Prove that this introduces an order on the class of all sets. Hint: Theorem of Shroeder-Bernstein proved in class is allowed to quote directly.

6. Show that $\sqrt[3]{17}$ is irrational.

7. Show that for rational numbers x, y, for a rational $z \neq 0$, and for positive rational ϵ, one has the following implication: if $|x - y| < \epsilon$ then $|xz - yz| < \epsilon|z|$.