HOME WORK III, ANALYSIS I

Due September 26.

A. In the next group of questions, please formulate the converse of each statement – i.e. the statement constituting that the given statement is false. Examples:

1. All apples are green

Converse: There exists an apple which is not green

2. For every $q \in \mathbb{Q}$ there exists an $n \in \mathbb{N}$ such that $q > n$.

Converse: There exists a $q \in \mathbb{Q}$ such that for every $n \in \mathbb{N}$ one has $q \leq n$.

3. $\exists z \in \mathbb{Z} : \forall a \in \{1, 2, 3\}$, $either$ $z + a \notin \mathbb{N}$, or $z|a$.

Converse: $\forall z \in \mathbb{Z} : \exists a \in \{1, 2, 3\} s.t.\ z + a \in \mathbb{N} and\ a \mod z \neq 0$.

Note: it is not asked if the statements are true or false; it is not asked what statements do they follow from or entail; it is merely asked to formulate converse statements.

1. In Brazil there is a lot of wild monkeys

Converse:

2. In Africa there is an elephant

Converse:

3. Every elephant in Africa has eaten at least three bananas

Converse:

4. If a cat has stripes, then its kittens have stripes

Converse:

5. $\forall n \in \mathbb{N}$, $n \in \mathbb{Z}$

Converse:

6. $\exists x \in \mathbb{Q} : x > 5$

Converse:

7. $\forall x \in \mathbb{N} : \exists q \in \mathbb{Q}, m \in \mathbb{Z} : x = qm$
Converse:

8. $\exists a \in [0,1] : \forall x > 3 : x + a \leq x - a$
Converse:

9. $\forall \epsilon > 0 \, \exists N \in \mathbb{N} : \frac{1}{N} < \epsilon$
Converse:

10. $\forall a \in \mathbb{Z}$ s.t. $2|a \, \exists b, c \in \mathbb{N} : a = bc$
Converse:

11. $\exists \epsilon < 0 : \forall x \in \mathbb{Z} : x - \epsilon \in \mathbb{Q}$
Converse:

12. $\forall \epsilon > 0 : \exists N \in \mathbb{N} : \forall n \geq N : |\frac{n}{n+1} - 1| \leq \epsilon$
Converse:

13. $\exists \epsilon > 0 : \forall \delta > 0 : \delta^2 > \epsilon$
Converse:

14. $\forall \epsilon > 0 : \exists \delta > 0 : \frac{1}{|\sin \epsilon|} < \delta$
Converse:

15. $\exists N \in \mathbb{N} : \forall \epsilon > 0 : \frac{N}{\epsilon} \in [1,5]$
Converse:

16. $\forall \epsilon > 0 \, \exists N \in \mathbb{N} : \forall n \geq N, |\frac{n^2}{3n^3 - n}| < \epsilon$
Converse:

17. $\forall \epsilon > 0$ either $\epsilon < 1$ or $\epsilon \geq 2$
Converse:

18. $\exists x \in \mathbb{Q} : \exists \epsilon > 0 : \forall \delta > 0, |\frac{(x+\epsilon)^2+1}{(x+\epsilon)^2-1} - \frac{x^2+1}{x^2-1}| > \delta$
Converse:

19. $\forall \epsilon > 0 : \exists N \in \mathbb{N} : \forall n \geq N, \forall m \geq N : |\frac{1}{n^2-n} - \frac{1}{m^2-m}| < \epsilon$
Converse:

20. $\exists \epsilon > 0 : \forall N \in \mathbb{N} : \exists n \geq N, \exists m \geq N : |(-1)^n - (-1)^m| \geq \epsilon$
Converse:

21. $\forall \epsilon > 0 \; \exists N \in \mathbb{N} : \forall n \geq N, |\frac{1}{n} - \frac{1}{n^2}| < \epsilon$
Converse:

22. $\exists \epsilon > 0 \; \forall N \in \mathbb{N} : \exists n \geq N, |\frac{1}{n^3} - \frac{n^2-1}{n^2+n}| \geq \epsilon$
Converse:

23. $a > b \implies \forall c \in \mathbb{Q} \forall d \in \mathbb{Q} : \; either \; c > bd \; or \; c \leq ad$
Converse:

24. $\exists a > 0, \exists b > 0 : \frac{1}{a} + \frac{1}{b} \leq \frac{2}{a+b}$
Converse:

25. $\forall \epsilon > 0 : \forall a > 0 \exists n \in \mathbb{N} : |a - \frac{1}{n}| \leq a + \epsilon$
Converse:

B. From the collection $\mathbb{Z} \cap [1, 25] \setminus \{1, 2, 3, 4, 5, 6, 14, 18\}$ select three statements which are true, and prove them.

C. From the collection $\mathbb{Z} \cap [1, 25] \setminus \{1, 2, 3, 4, 5, 6, 14, 18\}$ select three statements which are false, and disprove them.