

HOME WORK V, DIFFERENTIAL GEOMETRY: RIEMANNIAN CONNECTIONS AND GEODESICS, EXPONENTIAL MAP, CURVATURE TENSOR.

Due March 14. The Home Work must be uploaded on Canvas as a pdf. In addition to the class notes, you may find helpful Do Carmo's Riemannian geometry (Chapters 3 and 4), Chavel's Riemannian Geometry: a modern introduction (Chapters 1.4-1.7) and Ghomi's lecture notes 14, 15. Please contact me if you have any questions!

1. Using the expression of a connection in Christofel symbols, prove that a connection is symmetric if and only if $\Gamma_{ij}^k = \Gamma_{ji}^k$, for all i, j, k .
2. Compute the Christofel symbols for the Riemannian connection in the hyperbolic space (in class, we started by computing a few of them; you may use the first fundamental form which we computed.)
3. Prove that the curvature tensor is linear in each variable (it takes three vector fields as variables).