HOME WORK IV, HIGH-DIMENSIONAL GEOMETRY AND PROBABILITY, SPRING 2018

Due March 13. All the questions marked with an asterisk(s) are optional. The questions marked with a double asterisk are not only optional, but also have no due date.

Question 1**.** Show that if $2 < p < q < \infty$, then

$$d_{BM}(B_p^n, B_q^n) = n^{\frac{1}{p}-\frac{1}{q}}.$$

Hint 1: Use Holder's inequality for upper bound;

Hint 2: Use the result about the distance between the ball and the cube, along with the multiplicative triangle inequality for the Banach-Mazur distance, to get the estimate from below.

Question 2**.** Show that when K and L are symmetric, then

$$\inf_{x\in\mathbb{R}^n, T\in GL_n}\{\lambda > 0 : \; K \subset TL + x \subset \lambda K\} = \inf_{T\in GL_n}\{\lambda > 0 : \; K \subset TL \subset \lambda K\}.$$

Question 3**.** Modify the proof of John's theorem to conclude that the distance between the euclidean ball and any convex body in $\mathbb{R}^n$ (not necessarily symmetric) does not exceed n.

Question 4**.** Show that $d_{BM}(K, L) = d_{BM}(K^o, L^o)$.

Question 5^***.** Show that the Banach-Mazur distance between the simplex and the ball is n.

Question 6^{**}**.** Note that Question 3, together with the "triangle" inequality for Banach-Mazur distance, implies that for any pair of convex bodies K and L (not necessarily symmetric), $d_{BM}(K, L) \leq n^2$. Find any way to improve this estimate.