My research focuses on High-Dimensional Phenomena and Convexity — an exciting area at the crossroads of Analysis, Probability, and Geometry, with deep connections to Theoretical Computer Science, Information Theory, Statistics, Combinatorics, and beyond. The interplay of methods from these diverse areas has driven remarkable progress in recent years and initiated a golden age.

Convexity, isoperimetry, local versions of functional inequalities, symmetry, and more

For me, *convexity* is the property naturally exhibited by a large class of objects. For instance, the celebrated Brunn-Minkowski inequality states that the Lebesgue volume in a Euclidean space, denoted $|\cdot|$, is log-concave with respect to Minkowski addition of sets:

$$|\lambda A + (1-\lambda)B| \geq |A|^{\lambda}|B|^{1-\lambda}. \tag{1}$$

This is an indispensable tool in High-dimensional analysis, and more generally, many related *convexity principles* allow for deeper understanding of various phenomena, such as concentration of measure and isoperimetry. A convexity principle is a statement of the type "$\mathcal{F}(a)$ is concave", where $\mathcal{F}$ is some functional and a is from some linear space: for example, as per (1), $\mathcal{F}(K) = \log|K|$ is concave on the space of Borel sets with respect to Minkowski addition and scalar multiplication. Other relevant functionals are concave on certain subsets of appropriate functional spaces. Under right circumstances, "$\mathcal{F}(a)$ is concave" is equivalent to each of the three inequalities: for $t \in [0,1]$,

$$\mathcal{F}((1-t)a + tb) \geq (1-t)\mathcal{F}(a) + t\mathcal{F}(b); \tag{2}$$

$$\frac{d}{dt}\left(\mathcal{F}((1-t)a + tb) - (1-t)\mathcal{F}(a) - t\mathcal{F}(b)\right)_{t=0} \geq 0; \tag{3}$$

$$\frac{d^2}{dt^2}\mathcal{F}((1-t)a + tb)_{t=0} \leq 0. \tag{4}$$

Each of the three "steps" in this ladder is connected to its own collection of methods of proof, interpretations and applications, and together they comprise a rich theory. The inequalities (3) and (4) are called *local versions* of (2). In 2023/2024, I designed and taught two semesters of topics courses at Georgia Tech which included novel insights and connections between various convexity principles and isoperimetry, and recently I gave two invited short lecture courses on the topic, and developed lecture notes.

A number of questions have arisen in recent years concerning the role of symmetry and convexity in the isoperimetric-type inequalities, and the potential improvements of such inequalities under structural assumptions. Philosophical reason to expect isoperimetry to improve under symmetry assumptions is the fact that spectral gap is bigger when restricting to even functions; think, for instance, how the Poincaré inequality on the circle improves when one doesn't need to deal with the first term in the Fourier series of the function. One of the most famous questions in this spirit is the celebrated Log-Brunn-Minkowski conjecture of Böröczky, Lutwak, Yang, Zhang [18]. Jointly with Colesanti and Marsiglietti [27], I initiated the study of this conjecture via its *local* or *infinitesimal version*, which has subsequently become an active research direction in convexity, see e.g. [77, 52, 48, 105, 24]. In [93], we showed that the Log-BM conjecture implies the Dimensional Brunn-Minkowski conjecture of Gardner and Zvavitch [45], which states that an even log-concave measure is $\frac{1}{n}$-concave with respect to the Minkowski addition of symmetric convex sets. This was further extended in my work [52]. In [70] we made significant progress towards the Dim-BM conjecture: we showed that *for the standard Gaussian measure γ on $\mathbb{R}^n$, for any $\lambda \in [0,1]$ and any two convex sets K and L* ***containing the origin,***

$$\gamma(\lambda K + (1-\lambda)L)^{\frac{1}{2n}} \geq \lambda\gamma(K)^{\frac{1}{2n}} + (1-\lambda)\gamma(L)^{\frac{1}{2n}}. \tag{5}$$

Building up on this work, Eskenazis and Moschidis [36] showed the conjectured $1/n-$concavity of the Gaussian measure with respect to the addition of convex sets under the stronger assumption of origin-symmetry, which was extended by Cordero-Erausquin and Rotem [33] to all rotation-invariant log-concave measures.

In [92] I was able to show an absolute bound for the Dim-BM conjecture [92], valid for all even log-concave measures, which constitutes the first universal estimate of this kind: *for each $n \geq 1$, for any even log-concave probability measure μ on $\mathbb{R}^n$, for all symmetric convex sets K and L, and any $\lambda \in [0,1]$, one has*

$$\mu(\lambda K + (1-\lambda)L)^{n^{-4-o(1)}} \geq \lambda\mu(K)^{n^{-4-o(1)}} + (1-\lambda)\mu(L)^{n^{-4-o(1)}}. \tag{6}$$

Here $n^{-4-o(1)}$ can be replaced with $n^{-4}(\log n)^{-1}$, following Klartag's progress on the KLS conjecture [64].

Non-round Blaschke-Santaló inequalities: a recent exploration

Another famous conjecture which is a consequence of the Log-BM conjecture (see [114, 115]) is the *B-conjecture*, attributed to Banaszczyk and popularized by Latała. It states that *for an* ***even*** *log-concave measure* μ *and a* symmetric *convex set* K, $\log \mu(e^t K)$ *is concave in* $t \in \mathbb{R}$. The B-conjecture was verified in the affirmative in the case of the standard Gaussian measure γ in [30] (this result is referred to as the B-theorem), and applications include [68], [14]. In a recent breakthrough work [33] the B-conjecture was proved for a large class of measures including log-concave rotation-invariant. In [51] we characterized equality cases in the B-theorem, and applied our results to establish uniqueness of a special position for convex bodies which was studied by Bobkov [14].

B-conjecture is linked to a conjectured strengthening of the Brascamp-Lieb inequality [23]. Motivated by the B-conjecture, in recent works [26, 72] we initiated a study of a new interesting *non-round* generalization of the Blaschke-Santaló inequality which implies a strengthening of the Brascamp-Lieb inequality for even functions. A highlight of this new direction is the following result of ours: *for any symmetric convex body* K, *and for* $p \geq 2$,

$$|K| \cdot \left(\int_{K^o} \left(\prod_{i=1}^{n} |x_i| \right)^{\frac{2-p}{p-1}} dx \right)^{p-1} \leq (p-1)^{(p-1)n} |B_p^n|^p. \tag{7}$$

I like this inequality, since it is unusual to come by an isoperimetric-type inequality with a *non-round* optimizer. The inequality (7) has an equivalent functional counterpart, in the more general scenario, and that inequality is what does the heavy-lifting when it comes to consequences: it implies a family of strengthened Brascamp-Lieb inequalities for symmetric functions, which we hope to use to tackle the B-conjecture: *for* $p \geq q \geq 2$, *and* $V(x) = \|x\|_q^p/p$, *letting* μ *be the probability measure with density* Ce^{-V}, *we have, for even admissible functions* f :

$$\int_{\mathbb{R}^n} f^2 d\mu - \left(\int_{\mathbb{R}^n} f d\mu \right)^2 \leq \frac{p-1}{p} \int_{\mathbb{R}^n} \langle (\nabla^2 V)^{-1} \nabla f, \nabla f \rangle d\mu. \tag{8}$$

In a sequel paper [72], we found a relation of this new avenue of research to the Log-BM conjecture, and managed to verify the local form of the Log-BM conjecture for B_p^n with $p \geq 1$, improving upon the past results [48, 77], where only the case $p \geq 2$ was known.

High-dimensional phenomena and Random Matrices

High-Dimensional Probability is one of the key players in Random matrix theory, and it is especially useful in the Non-Asymptotic setting: we suppose that matrix dimensions are large but fixed, and study the precise asymptotics of various relevant functionals, such as singular values, as these dimensions tend to infinity. Under the strong assumption of i.i.d. mean-zero, variance-one sub-Gaussian entries, the following small ball behaviour was obtained by Rudelson and Vershynin for the smallest singular value $\sigma_n(A)$ of an $n \times n$ matrix in [111]:

$$P\left(\sigma_n(A) < \frac{\epsilon}{\sqrt{n}} \right) \leq c\epsilon + e^{-c_1 n}. \tag{9}$$

More generally, for $N \times n$ matrices under the same assumptions, they showed in [112]:

$$P\left(\sigma_n(A) < \epsilon(\sqrt{N+1} - \sqrt{n}) \right) \leq (c\epsilon)^{N-n+1} + e^{-Cn}. \tag{10}$$

Having a likely lower bound for the smallest singular value allows to infer that the matrix is likely to be invertible, which is useful e.g. for various problems in Statistics and Theoretical Computer Science. An important theme is understanding *universality* of the random matrix theoretic results: *Suppose some explicit properties of a specific well-behaved random matrix (say, with i.i.d. Gaussian entries) are well understood. To what extent can those properties continue to hold for more general ensembles of random matrices, with minimal assumptions?*

In this circle of problems, high dimension is usually a friend rather than an enemy. For instance, when the large dimension corresponds to a large number of *independent* random variables, the underlying system behaves in a predictable orderly fashion; examples of this phenomenon include the Law of Large Numbers and the Central Limit Theorem. Curiously, in many situations, *convexity* can replace *independence* in high-dimensional phenomena. Klartag's central limit theorem for convex sets [60, 61] is a classical example, and the recent breakthrough solutions of the slicing problem [65, 12] and the thin shell conjecture [66] can be viewed in the similar light.

Very recently, together with Fernandez and Mui we initiated the study of the ensemble of random matrices whose entries are jointly log-concave and isotropic [38]. It appears that the assumption of convexity can replace the assumption of independence also in this instance, although some exciting questions remain open. In [38] we show *the optimal bounds (9), (10) in the following cases:*

- *The distribution of A is log-concave, isotropic and unconditional, and $N = n$;*
- *$N \geq n$ and the columns of A are independent, isotropic and log-concave;*
- *$N \geq 2n$ and A is log-concave and isotropic.*

Our work utilizes, among other things, the net construction based on the random rounding from my joint paper with Klartag [67], and the recent breakthrough resolution of the slicing problem [65], [12].

Earlier, in [89, 94], with my co-authors I developed tools to work with the novel ensemble of *inhomogeneous* random matrices whose entries could have "different sizes" – that is, different means and variances (although we tend to assume that $\mathbb{E}\|A\|_{HS}^2 \leq Cn^2$.) Inhomogeneous random matrices have been studied very actively lately, and some of these works build up on our ideas, see e.g. [116, 5]; see also an unrelated powerful approach [8, 21, 22, 9, 10]. Some of our ideas influenced other works such as [54, 55, 56, 57] and my joint paper with Litvak [84] about minimal dispersion.

In my work with Tikhomirov and Vershynin [94], which followed up my paper [89], the tight small ball behavior (9) of the smallest singular value of *inhomogeneous* random matrices with heavy tails was deduced. My student Manuel Fernandez obtained important follow up results in [37], and jointly with Max Dabagia (another Georgia Tech student who attended my topics course, where a relevant research question was posed as an optional homework) they improved upon and extended my estimate [89] for the smallest singular value of rectangular random matrices.

One of the key new tools in this avenue of research is the novel net construction from my paper [89], which utilizes something that I call *regularized Hilbert-Schmidt norm*, and uses the concept of random rounding, previously investigated in the context of nets in the my joint work with Klartag [67]. Specifically, in [89], a much more involved version of the following result was derived:

There exists a net $\mathcal{N} \subset \frac{3}{2}B_2^n \setminus \frac{1}{2}B_2^n$ of size $\#\mathcal{N} \leq 10^n$ such that for any $N \times n$ random matrix A with independent columns one has, with probability $1 - e^{-cn}$: for every $x \in \mathbb{S}^{n-1}$ there exists a $y \in \mathcal{N}$ such that $|A(x-y)|^2 \leq \frac{c\boldsymbol{E}\|A\|_{HS}^2}{n}$.

Below I outline a sample subset of my current and future research directions and projects.

Future work: Generalized Log-Sobolev and Generalized Bobkov inequalities

The Prékopa–Leindler inequality [104, 82] is a functional extension of the Brunn–Minkowski inequality. As was pointed at [31], it may be written as

$$\int e^{-(tf+(1-t)g)^*} \geq \left(\int e^{-f^*}\right)^t \left(\int e^{-g^*}\right)^{1-t}, \tag{11}$$

where $0 < t < 1$ and $f^*(y) = \sup_{x\in\mathbb{R}^n}\big(\langle x, y\rangle - f(x)\big)$ denotes the Legendre transform. In other words, $\log \int e^{-f^*}$ *is a concave functional on a reasonable space of functions (for which the corresponding integrals exist).*

Adopting (3) to this situation, we derive the following interesting result, which can be viewed as a functional version of the Minkowski first inequality, and a generalization of the Log-Sobolev inequality: for any log-concave function ϕ and convex function G with $\int e^{-G} = 1$,

$$\int G^*\left(-\frac{\nabla\phi}{\phi}\right)\phi \geq n\int \phi + Ent(\phi).$$

Similar advances were previously done by Bobkov and Ledoux [15] in a partial case.

Inserting the celebrated Ehrhard inequality [34] into the same scheme in place of Prekopa-Leindler, we obtain, for all convex G and for all functions $h : \mathbb{R}^n \to [0,1]$ (such that the integrals make sense)

$$\int G^*\left(-\frac{\nabla h}{I(h)}\right) \cdot I(h) d\gamma \geq I\left(\int h d\gamma\right) \cdot \lim_{\lambda\to\infty} \frac{\Phi^{-1}\left(\int \Phi\left(-\lambda G\left(\frac{x}{\lambda}\right)\right) d\gamma\right)}{\lambda}. \tag{12}$$

Here Φ is the Gaussian cdf and I is the Gaussian isoperimetric profile. Plugging in $G(x) = -\sqrt{1-|x|^2} \cdot 1^{\infty}_{B_2^n}$ we recover the celebrated Bobkov inequality [13]: $\int_{\mathbb{R}^n} \sqrt{I(h)^2 + |\nabla h|^2} \geq I\left(\int_{\mathbb{R}^n} h d\gamma\right)$, which is the functional extension of the Gaussian isoperimetric inequality. With Barthe, Cordero-Erausquin, Ivanisvili, we are currently working on understanding better the striking inequality (12), its implications, and notable partial cases, as well as the "Brascamp-Lieb" version of Ehrhard's inequality which stems from analyzing (4) in this context.

Future work: towards the B-conjecture via restricting our inequality

One may show that the B-conjecture for the measure μ with density Ce^{-V}, where $V = \|x\|_q^p/p$, would follow from *a restricted version of* (8) on any symmetric convex body K. Therefore, a promising line of research is to extend (8) to this restricted version. One possibility is to utilize the techniques from mass transport (see e.g. Kolesnikov's work [69]). Another possibility is to find the L_2-proof of our results from [26], following the ideas in [33], as well as the even-odd function decomposition from our work [26].

Future work: The L_1-Mahler-type conjecture

The Blaschke-Santaló inequality has a famous counterpart: the celebrated Mahler Conjecture, which asks if the cube (as well as the cross-polytope and an entire special family of polytopes called Hanner polytopes) are the minimizers of the volume product $|K| \cdot |K^\circ|$. Numerous works are dedicated to this famous problem, see the survey [44]. Inspired by our work from [26], [72], it makes sense to consider an L_p-version of Mahler's conjecture. In the limit when $p \to 1$, we get the following curious question: *Which symmetric convex K minimizes $|K^\circ| \cdot \sup_{x\in K} \prod_{i=1}^n |x_i|$?* We conjecture that the answer is $K = B_1^n$. Jointly with Artstein-Avidan, Mui, Sadovsky and Slomka we are able to prove it in several key partial cases. Beyond that, it is interesting to understand $\sup_{x\in K} \prod_{i=1}^n |x_i|$, its properties, minimizing positions, and we hope to enrich the theory of convexity with several novel insights.

Future work: Isotropic Log-concave and inhomogeneous random matrices

The next question on the road following our recent work [38] is: *establish the lower bound for the smallest singular value of square isotropic log-concave random matrices, without the assumption of unconditionality.* In our work [38], all the steps work without the assumption of unconditionality except bounding the distance from a column of the matrix to the span of the remaining columns. In the unconditional case, this amounts to the small ball estimate for an isotropic log-concave vector, which is valid thanks to the recent breakthrough resolution of the slicing problem [65], however in general, conditioning a column on the realization of the remaining columns loses isotropicity (but not log-concavity), and we are unable to apply slicing. We intend to work out a more refined analysis in order to handle the general case, building up on the tools we developed.

In addition, many other questions are now tractable as well as of interest for this ensemble of isotropic log-concave random matrices: *find upper and lower bounds on all the intermediate singular values of ILC random matrices, for all the aspect ratios; establish bounds on the invertibility of shifted ILC matrices; study the phenomena of no-gaps delocalization of the null-vectors and singular vectors for ILC matrices.*

Regarding the inhomogeneous random matrices, my student Achintya Polavarapu is working on the following question, which in fact comprises many sub-questions: *estimate from above and below the small ball probability for intermediate singular values of an inhomogeneous random matrix.* In the case of the i. i. d. sub-Gaussian matrix, the relevant questions were studied in [103, 85, 118, 112]. The key ingredient in all these results is the so-called "Distance theorem". In the inhomogeneous setting, we obtained this in [94] in the partial case, and the general highly useful result was obtained by my student Manuel Fernandez [37], thereby making many questions in RMT related to inhomogeneous matrices tractable. In general, my long-term research goal is to further develop (together with students, postdocs and collaborators) the theory of inhomogeneous random matrices.

References

[1] R. Adamczak, A.E. Litvak, A. Pajor, N. Tomczak-Jaegermann: Quantitative estimates of the convergence of the empirical covariance matrix in log-concave ensembles, J. Amer. Math. Soc., 234(2010) 535-561.

[2] G. Aishwarya, D. Li, *Entropy and functional forms of the dimensional Brunn–Minkowski inequality in Gauss space,* preprint.

[3] G. Aishwarya, L. Rotem, *New Brunn–Minkowski and functional inequalities via convexity of entropy*, preprint.

[4] S. Alesker, S. Dar, V. D. Milman, *A Remarkable Measure Preserving Diffeomorphism between two Convex Bodies in R^n*, Geometriae Dedicata 74 (1999), 201–212.

[5] D. J. Altschuler, P. Oliveira Santos, K. Tikhomirov, P. Youssef, *On spectral outliers of inhomogeneous symmetric random matrices.* arXiv:2401.07852

[6] S. Artstein-Avidan, A. Giannopoulos, V.D. Milman, *Asymptotic geometric analysis. Part I*, Amer. Math. Soc., Providence, Rhode Island, 2015; xx+451 p.

[7] S. Artstein-Avidan, B. Klartag, V. D. Milman, *The Santaló point of a function, and a functional form of Santalo inequality*, Mathematika 51 (2004) 33–48

[8] A. Bandeira, M. Boedihardjo, R. van Handel, *Matrix concentration inequalities and free probability,* Invent. Math., (2023) 234:419–487.

[9] A. Bandeira, G. Cipolloni, D. Schröder, R. van Handel, *Matrix concentration inequalities and free probability II. Two-sided bounds and applications,* preprint.

[10] A. Bandeira, K. Lucca, P. Nizić-Nikolac, R. van Handel, *Matrix chaos inequalities and chaos of combinatorial type,* STOC2025, to appear.

[11] B. Berndtsson, V. Mastrantonis, Y. Rubinstein, *L^p-polarity, Mahler volumes, and the isotropic constant,* Analysis&PDE 17 (2024), 2179-2245.

[12] P. Bizeul, *The Slicing Conjecture via Small Ball Estimates*, arXiv:2501.06854 (2025)

[13] S. G. Bobkov, *An isoperimetric inequality on the discrete cube, and an elementary proof of the isoperimetric inequality in Gauss space,* The Annals of Probability 25, no. 1 (1997), 206–214.

[14] S. G. Bobkov, *On Milman's ellipsoids and M-position of convex bodies*, Concentration, functional inequalities and isoperimetry, Contemporary Mathematics 545, AMS (2011), 23-33

[15] S. Bobkov, M. Ledoux, *From Brunn-Minkowski to Brascamp-Lieb and to logarithmic Sobolev inequalities,* Geom. funct. anal. 10, 1028–1052 (2000).

[16] G. Bonnet, D. Dadush, U. Grupel, S. Huiberts, G. V. Livshyts *Combinatorial Diameter of Random Polytopes*, Volume 224, SoCG 2022.
`https://arxiv.org/pdf/2112.13027.pdf`

[17] K.J. Böröczky, P. Kalantzopoulos, *Log-Brunn-Minkowski inequality under symmetry*, Trans. Amer. Math. Soc. 375 (2022), 5987-6013.

[18] K.J. Böröczky, E. Lutwak, D. Yang, G. Zhang, *The log-Brunn-Minkowski-inequality*, Advances in Mathematics, 231 (2012), 1974-1997.

[19] K. J. Böröczky, E. Lutwak, D. Yang, G. Zhang, *The Logarithmic Minkowski Problem*, Journal of the American Mathematical Society, 26 (2013), 831-852.

[20] K. J. Böröczky, E. Lutwak, D. Yang, G. Zhang, *Affine images of isotropic measures*, J. Diff. Geom., 99 (2015), 407-442.

[21] T. Brailovskaya, R. van Handel, *Universality and sharp matrix concentration inequalities,* GAFA Volume 34, pages 1734–1838, (2024).

[22] T. Brailovskaya, R. van Handel, *Extremal random matrices with independent entries and matrix superconcentration inequalities,* Annals of Probability, to appear.

[23] H. Brascamp, E. Lieb, *On extensions of the Brunn–Minkowski and Prekopa–Leindler theorems, including inequalities for log-concave functions, and with an application to the diffusion equation*, J. Funct. Anal., Vol. 22, no.4 (1976) 366–389.

[24] S. Chen, Y. Huang, Q. Li, J. Liu, *The L_p-Brunn-Minkowski inequality for $p < 1$*, Advances in Mathematics 368 (2020), 107166.

[25] Y. Chen, *An Almost Constant Lower Bound of the Isoperimetric Coefficient in the KLS Conjecture*, Geom. Funct. Anal. 31, 34–61 (2021).

[26] A. Colesanti, A. Kolesnikov, G. Livschyts and L. Rotem *On weighted Blaschke–Santalo and strong Brascamp–Lieb inequalities*, arXiv:2409.11503.

[27] A. Colesanti, G. V. Livshyts, A. Marsiglietti, *On the stability of Brunn-Minkowski type inequalities*, Journal of Functional Analysis, Volume 273, 3, (2017), 1120-1139.
https://arxiv.org/pdf/1606.06586.pdf

[28] A. Colesanti, E. Francini, G. V. Livshyts, P. Salani, *The Brunn-Minkowski inequality for the first eigenvalue of the Ornstein-Uhlenbeck operator and log-concavity of the relevant eigenfunction,* submitted.

[29] D. Cordero-Erausquin, A. Eskenazis, *Concavity principles for weighted marginals,* preprint.

[30] D. Cordero-Erausquin, A. M. Fradelizi, B. Maurey, *The (B) conjecture for the Gaussian measure of dilates of symmetric convex sets and related problems*, Journal of Functional Analysis Vol. 214 (2004) 410-427.

[31] D. Cordero-Erausquin, B. Klartag, *Moment measures*, J. Functional Analysis, Vol. 268, No. 12, (2015), 3834-3866.

[32] D. Cordero-Erausquin, L. Rotem, *Several Results Regarding the (B)-Conjecture*, Geometric Aspects of Functional Analysis, Lecture Notes in Mathematics 2256 (2020), 247-262.

[33] D. Cordero-Erausquin, L. Rotem, *Improved log-concavity for rotationally-invariant measures of symmetric convex sets*, to appear in Annals of Probability. https://arxiv.org/abs/2111.05110

[34] A. Ehrhard, *Symetrisation dans l'espace de Gauss,* Math. Scand. 53 (1983), 281-301.

[35] R. Eldan, *Thin shell implies spectral gap up to polylog via a stochastic localization scheme*, Geometric and Functional Analysis: Volume 23, Issue 2 (2013), Page 532-569.

[36] A. Eskenazis, G. Moschidis, *The dimensional Brunn-Minkowski inequality in Gauss space*, Journal of Functional Analysis 280, no. 6 (2021), 108914.

[37] M. Fernandez, *A distance theorem for inhomogeneous random rectangular matrices*, preprint

[38] M. Fernandez, G. V. Livshyts, S. Mui, *On the smallest singular value of log-concave random matrices*, preprint.

[39] W. J. Firey, *Shapes of worn stones,* Mathematika, 21:1-11, 1974.

[40] M. Fradelizi, N. Gozlan, S. Sadovsky, S. Zugmeyer, *Transport-entropy forms of direct and Converse Blaschke-Santaló inequalities*, arXiv:2304.12839.

[41] M. Fradelizi, M. Meyer, *Some functional forms of Blaschke-Santaló inequality*, Math. Z. **256** no. 2, (2007), 379–395.

[42] M. Fradelizi, M. Meyer, *Increasing functions and inverse Santaló inequality for unconditional functions*, Positivity **12** no. 3 (2008), 407–420.

[43] M. Fradelizi, M. Meyer, *Some functional inverse Santaló inequalities*, Adv. Math. **218**, no. 5, (2008), 1430–1452.

[44] M. Fradelizi, M. Meyer, A. Zvavitch, *Volume product.* In: A. Koldobsky, A. Volberg (ed.) *Harmonic Analysis and Convexity.* Berlin, Boston: De Gruyter; 2023. p.163-222.

[45] R. Gardner, A. Zvavitch *Gaussian Brunn-Minkowski-type inequlities,* Trans. Amer. Math. Soc., 360, (2010), 10, 5333-5353.

[46] Q. Guan, *A note on Bourgain's slicing problem,* Preprint, arXiv:2412.09075, 2024.

[47] J. Haddad, D. Langharst, G. V. Livshyts, E. Putterman, *On the polar of Schneider's difference body,* submitted.

[48] R. van Handel, *The local logarithmic Brunn-Minkowski inequality for zonoids*, to appear in GAFA seminar notes.

[49] J. Hao, H. Huang, G. V. Livshyts, K. Tikhomirov, *Distribution of the minimal distance of random linear codes*, IEEE Transactions on Information Theory 68 (10), 6388-6401, (2022). `https://arxiv.org/abs/1912.12833`.

[50] O. Herscovici, G. V. Livshyts, *Kohler-Jobin meets Ehrhard: the sharp lower bound for the Gaussian principal frequency while the Gaussian torsional rigidity is fixed, via rearrangements,* Proc. Amer. Math. Soc. 152 (2024), 4437-4450.

[51] O. Herscovici, G. V. Livshyts, L. Rotem, A. Volberg, *Stability and the equality case in the B-theorem,* to appear in Israel Journal of Math.

[52] J. Hosle, A. V. Kolesnikov, G. V. Livshyts, *On the L_p-Brunn-Minkowski and dimensional Brunn-Minkowski conjectures for log-concave measures*,The Journal of Geometric Analysis 31 (6), 5799-5836, (2020).

[53] M. Ivaki, E. Milman, *L^p-Minkowski Problem Under Curvature Pinching*, International Mathematics Research Notices, Volume 2024, Issue 10, May 2024, Pages 8638–8652.

[54] V. Jain, A Sah, M Sawhney, *Singularity of discrete random matrices*, Geometric and Functional Analysis, 2021 - Springer.

[55] V. Jain, A. Sah, M. Sawhney, *On the smallest singular value of symmetric random matrices,* Combinatorics, Probability and Computing. 2022 Jul;31(4):662-83.

[56] V. Jain, A. Sah, M. Sawhney, *The smallest singular value of dense random regular digraphs*, International Mathematics Research Notices, Volume 2022, Issue 24, December 2022, Pages 19300–19334.

[57] V. Jain, S. Silwal, *A note on the universality of ESDs of inhomogeneous random matrices*, Latin American Journal of Probability and Mathematical Statistics (ALEA), vol. 18, 1047–1059 (2021).

[58] B. Jaye, G. V. Livshyts, G. Paouris, P. Pivovarov, *Minimizing Renyi entropy of the sum of independent random vectors,* IEEE Trans. Inform. Theory 66 (2020), no. 5, 2898–2903. 94A17 (60E15). `https://arxiv.org/pdf/1904.08038.pdf`

[59] B. Klartag, *Marginals of Geometric Inequalities*, Geometric Aspects of Functional Analysis, Lecture Notes in Mathematics 1910, Springer (2007), 133-166.

[60] B. Klartag, *A central limit theorem for convex sets*, Invent. Math., Vol. 168, (2007), 91-131.

[61] B. Klartag, *Power-law estimates for the central limit theorem for convex sets*, J. Funct. Anal., Vol. 245, (2007), 284-310.

[62] B. Klartag, *Poincare inequalities and moment maps,* Ann. Fac. Sci. Toulouse Math., Vol. 22, No. 1, (2013), 1–41.

[63] B. Klartag, *Logarithmically-concave moment measures I,* Geometric Aspects of Functional Analysis, Lecture Notes in Math. 2116, Springer (2014), 231–260.

[64] B. Klartag, *Logarithmic bounds for isoperimetry and slices of convex sets,* Ars Inveniendi Analytica (2023), Paper No. 4, 17 pp.

[65] B. Klartag, J. Lehec, *Affirmative Resolution of Bourgain's Slicing Problem using Guan's Bound*, Geom. Funct. Anal. 35, 1147–1168 (2025).

[66] B. Klartag, J. Lehec, *Thin-shell bounds via parallel coupling*, preprint.

[67] B. Klartag, G. V. Livshyts, *The lower bound for Koldobsky's slicing inequality via random rounding*, Israel Seminar (GAFA) 2017-2019 Volume II, (2020), p. 43-63. (20 pages) `https://arxiv.org/pdf/1810.06189.pdf`

[68] B. Klartag, R. Vershynin, *Small ball probability and Dvoretzky's Theorem*, Israel Journal of Math 157 (2007), 193-207.

[69] A. V. Kolesnikov, *Mass transportation and contractions* , arXiv:1103.1479

[70] A. V. Kolesnikov, G. V. Livshyts, *On the Gardner-Zvavitch conjecture: symmetry in the inequalities of Brunn-Minkowski type,* Advances in Mathematics 384 (2021) 107689. `https://arxiv.org/pdf/1807.06952.pdf`

[71] A. V. Kolesnikov, G. V. Livshyts, *On the Local version of the Log-Brunn-Minkowski conjecture and some new related geometric inequalities*, International Mathematics Research Notices, 142, 08 June 2021 `https://arxiv.org/pdf/2004.06103.pdf`

[72] A. Kolesnikov, G. Livshyts, L. Rotem *On weighted Blaschke–Santaló and strong Brascamp–Lieb inequalities*, arXiv:2409.11503.

[73] A.V. Kolesnikov, E. Milman, *Riemannian metrics on convex sets with applications to Poincaré and log-Sobolev inequalities*, Calc. Var. Partial Differential Equations, Vol. 55, (2016), 1–36.

[74] A.V. Kolesnikov, E. Milman, *Brascamp–Lieb-Type Inequalities on Weighted Riemannian Manifolds with Boundary*, J. Geom. Anal., (2017), vol. 27, no. 2, 1680–1702.

[75] A.V. Kolesnikov, E. Milman, *Poincaré and Brunn–Minkowski inequalities on the boundary of weighted Riemannian manifolds*, American Journal of Mathematics 140, no. 5 (2018), 1147-1185.

[76] A. V. Kolesnikov, E. Milman, *Sharp Poincare-type inequality for the Gaussian measure on the boundary of convex sets*, In: Klartag B., Milman E. (eds) Geometric Aspects of Functional Analysis. Lecture Notes in Mathematics, vol 2169. Springer, Cham, (2017), 221–234.

[77] A. V. Kolesnikov, E. Milman, *Local L_p-Brunn-Minkowski inequalities for $p < 1$*, Memoirs of the American Mathematical Society 277, no. 1360 (2022).

[78] A.V. Kolesnikov, E.M. Werner, *Blaschke–Santaló inequality for many functions and geodesic barycenters of measures*, Advances in Mathematics. 2022. v. 396. p. 108-110.

[79] R. Latała, *On some inequalities for Gaussian measures*, Proceedings of the International Congress of Mathematicians, Beijing, Vol. II, Higher Ed. Press, Beijing, 2002, pp. 813-822.

[80] Y. Lee, S. Vempala, *Eldan's Stochastic Localization and the KLS Hyperplane Conjecture: An Improved Lower Bound for Expansion*, FOCS 2017.

[81] Y. Lee, S. Vempala, *Slightly Improved Bound for the KLS Constant*, https://arxiv.org/pdf/2208.11644.pdf.

[82] L. Leindler, *On a certain converse of Hölder's inequality. II*, Acta Sci. Math. (Szeged) 33 (1972), no. 3-4, 217-223.

[83] J. Lehec, *A direct proof of the functional Santaló inequality*, C. R. Math. Acad. Sci. Paris **347** (2009), 55–58.

[84] A. E. Litvak, G. V. Livshys, *New bounds on the minimal dispersion*, Journal of Complexity (2022). https://arxiv.org/pdf/2108.10374.pdf

[85] A.E.Litvak, K.Tikhomirov, N.Tomczak-Jaegermann, *Small ball probability for the condition number of random matrices,* Geometric aspects of functional analysis. Vol. II, 125-137, Lecture Notes in Math., 2266, Springer, Cham (2020).

[86] G. V. Livshyts, *Maximal surface area of a convex set in R^n with respect to exponential rotation invariant measures*, Journal of Mathematical Analysis and applications, 404, (2013) 231-238. http://people.math.gatech.edu/~glivshyts6/livshyts_exponential.pdf.

[87] G. V. Livshyts, *Maximal surface area of a convex set in R^n with respect to log concave rotation invariant measures*, GAFA seminar notes, 2116, (2014), 355-383. http://arxiv.org/pdf/1308.4202v2.pdf.

[88] G. V. Livshyts, *Maximal surface area of a convex polytope in R^n with respect to log-concave rotation invariant measures*, Adv. Appl. Math., vol. 70, 54-69, (2015). http://people.math.gatech.edu/~glivshyts6/Livshyts_polytopes.pdf.

[89] G. V. Livshyts, *The smallest singular value of heavy-tailed not necessarily i.i.d. random matrices via random rounding,* Journal d'Analyse Mathematique, 145 (1), 257-306, (2021). https://arxiv.org/pdf/1811.07038.pdf

[90] G. V. Livshyts, *Some remarks about the maximal perimeter of convex sets with respect to probability measures,* Commun. Contemp. Math., (2020), https://doi.org/10.1142/S0219199720500376 https://arxiv.org/pdf/1904.06814.pdf

[91] G. V. Livshyts, *On a conjectural symmetric version of Ehrhard's inequality*, Trans. Amer. Math. Soc. 377 (2024), 5027-5085. https://arxiv.org/pdf/2103.11433.pdf

[92] G. V. Livshyts, *A universal bound in the dimensional Brunn-Minkowski inequality for log-concave measures*, Trans. Amer. Math. Soc. 376 (2023), 6663-6680. https://arxiv.org/pdf/2107.00095.pdf

[93] G. V. Livshyts, A. Marsiglietti, P. Nayar, A. Zvavitch, *On the Brunn-Minkowski inequality for general measures with applications to new isoperimetric type-inequalities,* Transactions of the AMS, 369 (2017), no. 12, 8725-8742. http://arxiv.org/abs/1504.04878.

[94] G. V. Livshyts, K. Tikhomirov, R. Vershynin, *The smallest singular value of inhomogeneous square random matrices*, Ann. Probab. 49 (2021), no. 3, 1286-1309. https://arxiv.org/abs/1909.04219

[95] A. Lytova, K. Tikhomirov, *On delocalization of eigenvectors of random non-Hermitian matrices*, Probab. Theory Relat. Fields 177, 465–524 (2020).

[96] A. Malliaris, J. Melbourne, C. Roberto, M. Roysdon, *Functional liftings of restricted geometric inequalities*, preprint.

[97] V. Mastrantonis, Y. Rubinstein, *Two-dimensional Blocki, L^p-Mahler, and Bourgain conjectures,* To appear in Indiana Univ. Math. J. 2024

[98] M. Meyer, A. Pajor, *On the Blaschke-Santaló inequality*, Arch. Math. 55 (1990), 82-93.

[99] E. Milman, *A sharp centro-affine isospectral inequality of Szegö-Weinberger type and the L^p-Minkowski problem*, to appear in J. Diff. Geom.

[100] E. Milman, *Centro-affine differential geometry and the log-Minkowski problem*, submitted

[101] E. Milman, J. Neeman, *The Gaussian Double-Bubble and Multi-Bubble Conjectures,* Annals of Math. 195, 89-206, 2022.

[102] E. Milman, J. Neeman, *The structure of isoperimetric bubbles in $\mathbb{R}^n$ and $\mathbb{S}^n$,* submitted.

[103] H. H. Nguyen, *Random matrices: overcrowding estimates for the spectrum*, J. Funct. Anal. 275 (2018), no. 8, 2197-2224. MR3841540

[104] A. Prékopa, *logarithmic concave measures with applications to stochastic programming*, Acta Sci. Math. (Szeged) 32 (1971), 301-316.

[105] E. Putterman, *Equivalence of the Local and Global Versions of the L^p-Brunn-Minkowski Inequality*, Journal of Functional Analysis 280, no. 9 (2021), 108956.

[106] E. Rebrova, K. Tikhomirov, *Coverings of random ellipsoids, and invertibility of matrices with i.i.d. heavy-tailed entries*, Israel J. Math. **227** (2018), no. 2, 507–544.

[107] L. Rotem, *A letter: The log-Brunn-Minkowski inequality for complex bodies*, https://arxiv.org/pdf/1412.5321.pdf.

[108] L. Rotem, *Surface area measures of log-concave functions*, Journal d'Analyse Mathématique (2022), appeared online at `https://doi.org/10.1007/s11854-022-0227-2`

[109] L. Rotem, *The anisotropic total variation and surface area measures*, to appear in Geometric Aspects of Functional Analysis. `https://arxiv.org/abs/2206.13146`

[110] M. Rudelson, *Invertibility of random matrices: norm of the inverse*, Annals of Mathematics 168 (2008), 575-600.

[111] M. Rudelson, R. Vershynin, *The Littlewood-Offord problem and invertibility of random matrices*, Adv. Math. 218 (2008), no. 2, 600-633.

[112] M. Rudelson, R. Vershynin, *Smallest singular value of a random rectangular matrix,* Communications on Pure and Applied Mathematics 62 (2009), 1707-1739.

[113] M. Rudelson, R. Vershynin, *Delocalization of eigenvectors of random matrices with independent entries,* Duke Math. J. Volume 164, Number 13 (2015), 2507-2538.

[114] C. Saroglou, *Remarks on the conjectured log-Brunn-Minkowski inequality*, Geom. Dedicata, (2015), Vol. 177, no. 1, 353–365.

[115] C. Saroglou, *More on logarithmic sums of convex bodies*, Mathematika 62, no. 3 (2016), 818-841.

[116] K. Tikhomirov, *On pseudospectrum of inhomogeneous non-Hermitian random matrices*. arXiv:2307.08211

[117] K. Tikhomirov, *Singularity of random Bernoulli matrices*, Ann. of Math. (2) **191** (2020), no. 2, 593–634.

[118] F. Wei, *Upper bound for intermediate singular values of random matrices*, Journal of Mathematical Analysis and Applications 445, no. 2 (2017), 1530-1547.