TEST 1, PROBABILITY I, FALL 2016

1. Given a probability space $(\Omega, \mathcal{F}, P)$, and a pair of Borel measurable functions $f, g : \Omega \to \mathbb{R}$, such that $f = g$ almost everywhere with respect to P, show that $\int f(\omega) dP(\omega) = \int g(\omega) dP(\omega)$. (please follow all the necessary steps from the definition of integral)

2. Let X be a Gaussian random variable. Please estimate from above and below $P(X \leq 10)$.

3. Given a probability space $(\Omega, \mathcal{F}, P)$, and a random variable X with bounded fifth moment, prove that for $t > 0$

$$P(|X| < t) \geq 1 - \frac{\mathbb{E}|X|^5}{t^5}.$$

4. Let X be a random variable taking values on the interval $[1, 2]$. Find sharp lower and upper estimates on the quantity $\mathbb{E}X \cdot \mathbb{E}\frac{1}{X}$. Provide an example of a random variable for which the lower estimate is attained. Provide an example of a random variable for which the upper estimate is attained.

Hint. For the lower bound, justify and use the inequality

$$ab \leq \frac{1}{2}(\frac{a}{2} + b)^2.$$