

TEST 2, PROBABILITY I

Select 2 problems out of 3.

1. Show that if X_1, \dots, X_n, \dots are independent and have characteristic function $e^{-|t|^\alpha}$, then $\frac{X_1 + \dots + X_n}{n^{\frac{1}{\alpha}}}$ has the same distribution as X_1 .

2. Show that if X and Y are independent and $X + Y$ and X have the same distribution, then $Y = 0$ a.s.

Hint: use characteristic functions.

3. Suppose X_i are i.i.d. with $X_i \geq 0$, $\mathbb{E}X_i = 1$ and $\sigma^2 = 1$. Show that $2(\sqrt{X_1 + \dots + X_n} - \sqrt{n})$ converges weakly to the standard normal distribution.

Hint: Use the CLT and the definition of weak convergence (and don't forget that $F_X(x) = P(X \leq x)$).