Abstracts for the lecture courses

Shiri Artstein-Avidan, Tel Aviv University

Title: Dualities, measure concentration and transportation

Abstract: In these three lectures we shall review and discuss in detail the fascinating connection between duality transforms (mainly Legendre and polarity, but also those associated with other cost functions), measure transportation, cost sub-gradiant mappings and concentration inequalities. We will see how the Prékopa Leindler inequality and other analogues of it come up in this context and can be utilized to get more sophisticated concentration inequalities as well as a deeper understanding of the underlying geometry.

Alexander Koldobsky, University of Missouri-Columbia

Title: Fourier analysis in geometric tomography

Abstract: Geometric tomography is the study of geometric properties of solids based on data about sections and projections of these solids. The lectures will include:

1. An outline of proofs of two of the main features of the Fourier approach to geometric tomography - the relation between the derivatives of the parallel section function of a body and the Fourier transform (in the sense of distributions) of powers of the norm generated by this body, and the Fourier characterization of intersection bodies.

2. The Busemann-Petty problem asks whether symmetric convex bodies with uniformly smaller areas of central hyperplane sections necessarily have smaller volume. We will prove an isomorphic version of the problem with a constant depending on the distance from the class of intersection bodies. This will include a generalization to arbitrary measures in place of volume.

3. The slicing problem of Bourgain asks whether every symmetric convex body of volume one has a hyperplane section with area greater than an absolute constant. We will consider a version of this problem for arbitrary measures in place of volume. We will show that the answer is affirmative for many classes of bodies, but in general the constant must be of the order $1/\sqrt{n}$.

4. Optimal estimates for the maximal distance from a convex body to the classes of intersection bodies and the unit balls of subspaces of L_p.

5. We will use the Fourier approach to prove that the only polynomially integrable convex bodies, i.e. bodies whose parallel section function in every direction is a polynomial of the distance from the origin, are ellipsoids in odd dimensions.

Grigoris Paouris, Texas A&M

Title: Concentration and Convexity

Abstract: The Concentration of measure phenomenon is a fundamental tool of high dimensional probability and of Asymptotic Geometric Analysis. Independence or Isoperimetry are two typical reasons for the appearance of this phenomenon. In these talks I will introduce the phenomenon and I will show how High dimensional Geometry affects the concentration. In particular I will explain how "convexity" can be used to establish strong concentration inequalities in the Gauss space and how the "convexity" of the underline measure is responsible for deviation principles.

Elisabeth Werner, Case Western Reserve University

Title: Floating body, Approximation by polytopes and Data depth

Abstract: Two important closely related notions in affine convex geometry are the floating body and the affine surface area of a convex body. The floating body of a convex body is obtained by cutting off caps of volume less or equal to a fixed positive constant. Taking the right-derivative of the volume of the floating body gives rise to an affine invariant, the affine surface area. This was established for all convex bodies in all dimensions by Schuett and Werner. There is a natural inequality associated with affine surface area, the affine isoperimetric inequality, which states that among all convex bodies, with fixed volume, affine surface area is maximized for ellipsoids.

Due to its important properties, which make them effective and powerful tools, affine surface area and floating body are omnipresent in geometry and have applications in many other areas of mathematics, e.g., in problems of approximation of convex bodies by polytopes and for the notion of halfspace depth for multivariate data from statistics.