

HOME WORK 3, PROBABILITY II. BROWNIAN MOTION.

1. Let $B(t)$ be a linear Brownian motion. Show that almost surely there exists $t \geq 0$ with $D^*B(t) = 0$.

2. Let $B(t)$ be a linear Brownian motion. Find two stopping times T and S with $S \leq T$, $\mathbb{E}S < \infty$ and $\mathbb{E}(B(S)^2) \geq \mathbb{E}(B(T)^2)$.

3. Let $B(t)$ be a linear Brownian motion. Show that for $\sigma > 0$, the process

$$e^{\sigma B(t) - \frac{\sigma^2 t}{2}}$$

is a martingale.